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Daniel Sitaru

ALGEBRAIC PHENOMENON

ÎNVĂȚARE DE EXCELENȚĂ®
supersucces

Editura Paralela 45

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Chapter 1 Famous Theorems

CAUCHY-SCHWARZ's-Inequality

$$(ax+by)^2 \le (a^2+b^2)(x^2+y^2);\ a,b,x,y \in \mathbb{R}$$

$$(ax+by+cz)^2 \le (a^2+b^2+c^2)(x^2+y^2+z^2);\ a,b,c,x,y,z \in \mathbb{R}$$

$$\left(\sum_{i=1}^{n} a_i x_i\right)^2 \le \left(\sum_{i=1}^{n} a_i^2\right)\left(\sum_{i=1}^{n} x_i^2\right); a_i, x_i \in \mathbb{R}, i \in \overline{1,n}$$

$$\frac{a^2}{x}+\frac{b^2}{y} \ge \frac{(a+b)^2}{x+y}; a,b \in \mathbb{R}; x,y \in (0,\infty)$$

$$\frac{a^2}{x}+\frac{b^2}{y}+\frac{c^2}{z} \ge \frac{(a+b+c)^2}{x+y+z}; a,b,c \in \mathbb{R}; x,y,z \in (0,\infty)$$

$$\frac{a}{x}+\frac{b}{y}+\frac{c}{z} \ge \frac{(a+b+c)^2}{ax+by+cz}; a,b,c,x,y,z \in (0,\infty)$$

$$\frac{a_1^2}{x_1}+\frac{a_2^2}{x_2}+\cdots+\frac{a_n^2}{x_n} \ge \frac{(a_1+a_2+\cdots+a_n)^2}{x_1+x_2+\cdots+x_n}; a_i \in \mathbb{R}; x_i > 0; i \in \overline{1,n}$$

$$\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b} \ge \frac{3}{2}; (\forall) a,b,c \in (0,\infty)$$

$$\frac{a}{b+nc}+\frac{b}{c+na}+\frac{c}{a+nb} \ge \frac{3}{n+1}; a,b,c \in (0,\infty);\ n \in \mathbb{N}^*$$

MINKOWSKI's-Inequality

$$\sqrt{(x+a)^2+(y+b)^2} \le \sqrt{x^2+y^2}+\sqrt{a^2+b^2}$$

$$\sqrt{(x+y+z)^2+(a+b+c)^2} \le \sqrt{x^2+a^2}+\sqrt{y^2+b^2}+\sqrt{z^2+c^2}$$

$$\sqrt{(x+a)^2+(y+b)^2+(z+c)^2} \le \sqrt{x^2+y^2+z^2}+\sqrt{a^2+b^2+c^2}$$

$$\sqrt{(x_1+a_1)^2+(x_2+a_2)^2+\cdots+(x_n+a_n)^2}$$
$$\le \sqrt{x_1^2+x_2^2+\cdots+x_n^2}+\sqrt{a_1^2+a_2^2+\cdots+a_n^2}$$
$$x_i; a_i \in \mathbb{R}; i \in \overline{1,n}; n \in \mathbb{N}^*$$

$$\left(\sum_{i=1}^{n} |x_i+y_i|^p\right)^{\frac{1}{p}} \le \left(\sum_{i=1}^{n} |x_i|^p\right)^{\frac{1}{p}} + \left(\sum_{i=1}^{n} |y_i|^p\right)^{\frac{1}{p}}$$
$$p > 1; x_i, y_i \in \mathbb{R}; i \in \overline{1,n}; n \in \mathbb{N}^*$$

Chapter 2 Questions

1. Find the minimum of the expression:

$$E = \frac{4ab - 11a^2 - 14b^2}{3(a^2 + b^2)}; a, b \in \mathbb{R}^*$$

2. Prove that if $a, b, c \in (0, \infty)$ then:

$$\sum a^2 \left(\frac{1}{2016a^2 + 2015b^2} + \frac{1}{2016a^2 + 2015b^2}\right) > \frac{3}{4031}$$

3. Prove that if $a, b, c \in (0, \infty); m \in \mathbb{R}^*, a + b + c = \frac{2}{m^2}$ then:

$$\sqrt{m^2a + 1} + \sqrt{m^2b + 1} + \sqrt{m^2c + 1} \leq 4$$

4. Prove that if $a, b, c \in \left(\frac{1}{k}, \infty\right); k > 0, \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 2k$ then:

$$\sqrt{ka - 1} + \sqrt{kb - 1} + \sqrt{kc - 1} \geq \sqrt{a + b + c}$$

5. Prove that if $x, y, z \geq 0; t \in \mathbb{R}; n \in \mathbb{N}, x + y + z = 6t$ then:

$$x\sqrt{y + n} + y\sqrt{z + n} + z\sqrt{x + n} \leq \sqrt{3}\left(6t^2 + (3n + 1)t\right)$$

6. Prove that if $x, y, z \geq 0$ și $x + y + z = 2$ then:

$$x\sqrt{y + 1} + y\sqrt{z + 1} + z\sqrt{x + 1} \leq 2\sqrt{3}$$

7. Prove that if $a, b, c, d \in (0, \infty); \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{d}$ then:

$$\frac{(a + b)(b + c)(c + a)}{(a - d)(b - d)(c - d)} \geq 27$$

8. Prove that if $a, b, c \in (0, \infty)$ then:

$$\left(\frac{1}{a} + \frac{1}{b}\right)\left(\frac{1}{b} + \frac{1}{c}\right)\left(\frac{1}{c} + \frac{1}{a}\right) + \frac{1}{abc} \geq \frac{81}{(a + b)(b + c)(c + a) + abc}$$

Chapter 3 Solutions

1. Find the minimum of the expression:

$$\boldsymbol{E=\frac{4ab-11a^2-14b^2}{3(a^2+b^2)};a,b\in\mathbb{R}^*}$$

Proof:

$$E=\frac{4ab-11a^2-14b^2}{3(a^2+b^2)}=\frac{4a^2+b^2+4ab-15a^2-15b^2}{3(a^2+b^2)}=$$

$$=\frac{(2a+b)^2-15(a^2+b^2)}{3(a^2+b^2)}=\frac{(2a+b)^2}{3(a^2+b^2)}-5\geq -5,\quad \min E=-5\ for\ 2a+b=0$$

2. Prove that if: $\boldsymbol{a,b,c\in(0,\infty)}$ **then:**

$$\boldsymbol{\sum a^2\left(\frac{1}{2016a^2+2015b^2}+\frac{1}{2016a^2+2015b^2}\right)>\frac{3}{4031}}$$

Proof:

$$\frac{a^2}{2016a^2+2015b^2}+\frac{b^2}{2016b^2+2015a^2}>\frac{(a+b)^2}{4031(a^2+b^2)}>\frac{1}{4031}\cdot\frac{(a+b)^2}{a^2+b^2}>\frac{1}{4031}$$

$$because\ \frac{(a+b)^2}{a^2+b^2}>1;\ (a+b)^2>a^2+b^2, a^2+2ab+b^2>a^2+b^2; 2ab>0$$

Analogous:

$$\frac{a^2}{2016a^2+2015c^2}+\frac{c^2}{2016c^2+2015a^2}>\frac{1}{4031}$$

$$\frac{b^2}{2016b^2+2015c^2}+\frac{c^2}{2016c^2+2015b^2}>\frac{1}{4031}$$

$$\sum a^2\left(\frac{1}{2016a^2+2015b^2}+\frac{1}{2016a^2+2015b^2}\right)>\frac{3}{4031}$$

3. Prove that if: $\boldsymbol{a,b,c\in(0,\infty); m\in\mathbb{R}^*, a+b+c=\frac{2}{m^2}}$ **then:**

$$\boldsymbol{\sqrt{m^2a+1}+\sqrt{m^2b+1}+\sqrt{m^2c+1}\leq 4}$$

Bibliography

1. D.M. Bătinețu-Giurgiu, Maria Bătinețu-Giurgiu, I. Bîrchi-Damian, A. Semenescu: *Real analysee*, Editura „Matrix – Rom", Bucharest, 2004.

2. Gh. Sireţchi: *Differential and Integral Calculus*, Scientific and Encyclopedic Publishing, Bucharest, 1985.

3. Dragoș Popescu, George Oboroceanu: *Exercises and Problems in Combinatorics and Number Theory Algebra*, Didactic and Pedagogical Publishing, Bucharest, 1979.

4. Gheorghe Andrei, Constantin Caragea, Gheorghe Bordea: *Algebra for Entrance Contests and School Olympiads*, Topaz Publishing, Constanța, 1993.

5. Gheorghe Andrei, Constantin Caragea, Viviana Ene: *Algebra for Entrance Contests and School Olympiads*, Scorpion Publishing, Bucharest, 1995.

6. Ion Savu & colaborators: *Preparatory Problems for School Olympiads*, Art Publishing, Bucharest, 2006.

7. M. Popescu, D. Sitaru: *"Traian Lalescu" Contest. Geometry Problems*, Lithography University of Craiova Publishing, Craiova, 1985.

8. Daniel Sitaru, Claudia Nănuţi: *National Contest of Applied Mathematics – "Adolf Haimovici" – the County Stage"*, Ecko – Print Publishing, Drobeta Turnu Severin, 2011.

9. Daniel Sitaru, Claudia Nănuţi: *National Contest of Applied Mathematics – "Adolf Haimovici" – the National Stage"*, Ecko – Print Publishing, Drobeta Turnu Severin, 2011.

10. Daniel Sitaru, Claudia Nănuți: *Contest Problems*, Ecko – Print Publishing, Drobeta Turnu Severin, 2011.

11. Daniel Sitaru, Claudia Nănuţi: *Baccalaureate – Problems – Solutions – Topics Scales*, Ecko – Print Publishing, Drobeta Turnu Severin, 2011.

12. Daniel Sitaru: *Affine and Euclidiane Geometry Problems*, Ecko – Print Publishing, Drobeta Turnu Severin, 2012.

13. Daniel Sitaru, Claudia Nănuţi: *Baccalaureate – Problems – Tests – Topics – 2010 – 2013*, Ecko – Print Publishing, Drobeta Turnu Severin, 2012.

14. Daniel Sitaru: *Hipercomplex and Quaternion Geometry*, Ecko – Print Publishing, Drobeta Turnu Severin, 2013.

15. Daniel Sitaru, Claudia Nănuţi: *Algebra Basis*, Ecko – Print Publishing, Drobeta Turnu Severin, 2013.

16. Daniel Sitaru, Claudia Nănuţi: *Mathematical Lessons*, Ecko – Print Publishing, Drobeta Turnu Severin, 2013.